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PERFECT CUBES.

BY PROF. W. D. HENKLE, SALEM, OHIO.

In my first article on this subject (See ANALYST Vol. I. No. 5.) I explained how the tens' figure of the root of a perfect cube can be obtained from the tens' figure of the cube. In this article I shall extend the discussion to the hundreds' figure of the root. In order not to use too much space in the ANALYST I adopt a brief mode of indicating the facts, which may be readily comprehended by a reference to the former article. The figures after the braces are the endings of the perfect cubes and the subs the endings of the roots. The figures in the upper rows before the braces represent the hundreds of the cube and those in the lower rows the corresponding hundreds of the root. The direction after each brace refers to the mode of obtaining any figure in the lower from the corresponding figure in the upper row. (I use for brevity the word *figure* instead of *number represented by the figure*). In each case of multiplication or of multiplication and addition the tens are to be cast out or the left-hand digit is to be rejected from the product or sum.

$$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\7&4&1&8&5&2&9&6&3&0\end{matrix}\right\}01_{01};\ \text{multiply by 7 and cast out the tens.}$$

$$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\4&1&8&5&2&9&6&3&0&7\end{matrix}\right\}\begin{matrix}11_{71}\\21_{41}\\81_{61}\end{matrix};\ \text{mult. by 7, add 7, and cast out the tens.}$$

$$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\6&3&0&7&4&1&8&5&2&9\end{matrix}\right\}31_{11};\ \text{mult. by 7, add 9, and cast out the tens.}$$

$$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\9&6&3&0&7&4&1&8&5&2\end{matrix}\right\}41_{81};\ \text{mult. by 7, add 2, and cast out the tens.}$$

$$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\5&2&9&6&3&0&7&4&1&8\end{matrix}\right\}51_{51};\ \text{mult. by 7, add 8, and cast out the tens.}$$

$$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\3&0&7&4&1&8&5&2&9&6\end{matrix}\right\}61_{21};\ \text{mult. by 7, add 6, and cast out the tens.}$$

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\2&9&6&3&0&7&4&1&8&5\end{matrix}\right\}71_{91};$ × by 7, add 5, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\8&5&2&9&6&3&0&7&4&1\end{matrix}\right\}91_{31};$ × by 7, add 1, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\8&1&4&7&0&3&6&9&2&5\end{matrix}\right\}\begin{matrix}03_{87}\\33_{77}\end{matrix};$ × by 3, add 5, and cast out the tens.

$\left.\begin{matrix}1&1&3&4&5&6&7&8&9&0\\6&9&2&5&8&1&4&7&0&3\end{matrix}\right\}13_{17};$ × by 3, add 3, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\9&2&5&8&1&4&7&0&3&6\end{matrix}\right\}23_{47};$ × by 3, add 6, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\4&7&0&3&6&9&2&5&8&1\end{matrix}\right\}43_{07};$ × by 3, add 1, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\5&8&1&4&7&0&3&6&9&2\end{matrix}\right\}\begin{matrix}53_{37}\\73_{97}\\83_{27}\end{matrix};$ × by 3, add 2, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\2&5&8&1&4&7&0&3&6&9\end{matrix}\right\}63_{67};$ × by 3, add 9, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\0&3&6&9&2&5&8&1&4&7\end{matrix}\right\}93_{57};$ × by 3, add 7, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\8&1&4&7&0&3&6&9&2&5\end{matrix}\right\}07_{43};$ × by 3, add 5, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\3&6&9&2&5&8&1&4&7&0\end{matrix}\right\}\begin{matrix}17_{73}\\27_{03}\\47_{63}\end{matrix};$ × by 3, add 0, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\6&9&2&5&8&1&4&7&0&3\end{matrix}\right\}37_{33};$ × by 3, add 3, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\4&7&0&3&6&9&2&5&8&1\end{matrix}\right\}57_{93};$ × by 3, add 1, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\0&3&6&9&2&5&8&1&4&7\end{matrix}\right\}\begin{matrix}67_{23}\\97_{13}\end{matrix};$ × by 3, add 7, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\9&2&5&8&1&4&7&0&3&6\end{matrix}\right\}77_{53};$ × by 3, add 6, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\2&5&8&1&4&7&0&3&6&9\end{matrix}\right\}87_{83};$ × by 3, add 9, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\2&9&6&3&0&7&4&1&8&5\end{matrix}\right\}09_{69},$ × by 7, add 5, and cast out the tens.

$\left.\begin{matrix}1&2&3&4&5&6&7&8&9&0\\6&3&0&7&4&1&8&5&2&9\end{matrix}\right\}\begin{matrix}19_{39}\\79_{59}\\89_{29}\end{matrix};$ × by 7, add 9, and cast out the tens.

$$\left.\begin{array}{cccccccccc}1&2&3&4&5&6&7&8&9&0\\8&5&2&9&6&3&0&7&4&1\end{array}\right\}29_{09}; \times \text{ by 7, add 1, and cast out the tens.}$$

$$\left.\begin{array}{cccccccccc}1&2&3&4&5&6&7&8&9&0\\7&4&1&8&5&2&9&6&3&0\end{array}\right\}39_{79}; \times \text{ by 7, add 0, and cast out the tens.}$$

$$\left.\begin{array}{cccccccccc}1&2&3&4&5&6&7&8&9&0\\5&2&9&6&3&0&7&4&1&8\end{array}\right\}49_{49}, \times \text{ by 7, add 8, and cast out the tens.}$$

$$\left.\begin{array}{cccccccccc}1&2&3&4&5&6&7&8&9&0\\1&8&5&2&9&6&3&0&7&4\end{array}\right\}59_{19}; \times \text{ by 7, add 4, and cast out the tens.}$$

$$\left.\begin{array}{cccccccccc}1&2&3&4&5&6&7&8&9&0\\4&1&8&5&2&9&6&3&0&7\end{array}\right\}69_{89}; \times \text{ by 7, add 7, and cast out the tens.}$$

$$\left.\begin{array}{cccccccccc}1&2&3&4&5&6&7&8&9&0\\3&0&7&4&1&8&5&2&9&6\end{array}\right\}99_{99}; \times \text{ by 7, add 6, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&1&3&5&7&9\\&3&9&5&1&7\\\text{or}&8&4&0&6&2\end{array}\right\}\begin{array}{l}12_{08}\\52_{78};\\92_{48}\end{array} \times \text{ by 3, add 0 or 5, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&1&3&5&7&9\\&5&1&7&3&9\\\text{or}&0&6&2&8&4\end{array}\right\}\begin{array}{l}12_{58}\\92_{98}\end{array}; \times \text{ by 3, add 2 or 7, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&2&4&6&8&0\\&7&3&9&5&1\\\text{or}&2&8&4&0&6\end{array}\right\}\begin{array}{l}32_{18}\\72_{38}\end{array}; \times \text{ by 3, add 1 or 6, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&2&4&6&8&0\\&9&5&1&7&3\\\text{or}&4&0&6&2&8\end{array}\right\}\begin{array}{l}32_{68}\\72_{88}\end{array}; \times \text{ by 3, add 3 or 8, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&1&3&5&7&9\\&6&2&8&4&0\\\text{or}&1&7&3&9&5\end{array}\right\}52_{28}; \times \text{ by 3, add 3 or 8, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&1&3&5&7&9\\&6&0&4&8&2\\\text{or}&1&5&9&3&7\end{array}\right\}04_{34}; \times \text{ by 2, add 4 or 9, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&1&3&5&7&9\\&3&7&1&5&9\\\text{or}&8&2&6&0&4\end{array}\right\}\begin{array}{l}04_{84}\\44_{14}\end{array}; \times \text{ by 2, add 1 or 6, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&2&4&6&8&0\\&8&2&6&0&4\\\text{or}&3&7&1&5&9\end{array}\right\}24_{24}; \times \text{ by 2, add 4 or 9, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&2&4&6&8&0\\&5&9&3&7&1\\\text{or}&0&4&8&2&6\end{array}\right\}24_{74}; \times \text{ by 2, add 1 or 6, and cast out the tens.}$$

$$\left.\begin{array}{rccccc}&1&3&5&7&9\\&5&9&3&7&1\\\text{or}&0&4&8&2&6\end{array}\right\}\begin{array}{l}44_{64}\\84_{44}\end{array}; \times \text{ by 2, add 3 or 8, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 2&4&6&8&0\\ & 4&8&2&6&0\\ \text{or} & 9&3&7&1&5\end{array}\right] 64_{04};\ \times \text{ by 2, add 0 or 5, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 2&4&6&8&0\\ & 6&0&4&8&2\\ \text{or} & 1&5&9&3&7\end{array}\right] 64_{54};\ \times \text{ by 2, add 2 or 7, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 1&3&5&7&9\\ & 2&6&0&4&8\\ \text{or} & 7&1&5&9&3\end{array}\right] 84_{94},\ \times \text{ by 2, add 0 or 5, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 2&4&6&8&0\\ & 5&9&3&7&1\\ \text{or} & 0&4&8&2&6\end{array}\right] 16_{06};\ \times \text{ by 2, add 1 or 6, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 2&4&6&8&0\\ & 7&1&5&9&3\\ \text{or} & 2&6&0&4&8\end{array}\right] \begin{array}{l}16_{56}\\56_{36}\end{array};\ \times \text{ by 2, add 3 or 8, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 1&3&5&7&9\\ & 6&0&4&8&2\\ \text{or} & 1&5&9&3&7\end{array}\right] 36_{46};\ \times \text{ by 2, add 4 or 9, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 1&3&5&7&9\\ & 3&7&1&5&9\\ \text{or} & 8&2&6&0&4\end{array}\right] 36_{96};\ \times \text{ by 2, add 1 or 6, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 2&4&6&8&0\\ & 4&8&2&6&0\\ \text{or} & 9&3&7&1&5\end{array}\right] \begin{array}{l}56_{86}\\96_{16}\end{array};\ \times \text{ by 2, add 0 or 5, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 1&3&5&7&9\\ & 2&6&0&4&8\\ \text{or} & 7&1&5&9&3\end{array}\right] 76_{26};\ \times \text{ by 2, add 0 or 5, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 1&3&5&7&9\\ & 4&8&2&6&0\\ \text{or} & 9&3&7&1&5\end{array}\right] 76_{76};\ \times \text{ by 2, add 2 or 7, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 2&4&6&8&0\\ & 6&0&4&8&2\\ \text{or} & 1&5&9&3&7\end{array}\right] 96_{66};\ \times \text{ by 2, add 2 or 7, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 2&4&6&8&0\\ & 6&2&8&4&0\\ \text{or} & 1&7&3&9&5\end{array}\right] \begin{array}{l}08_{02}\\88_{42}\end{array};\ \times \text{ by 3, add 0 or 5, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 2&4&6&8&0\\ & 8&4&0&6&2\\ \text{or} & 3&9&5&1&7\end{array}\right] \begin{array}{l}08_{52}\\48_{22}\\88_{92}\end{array};\ \times \text{ by 3, add 2 or 7, and cast out the tens.}$$

$$\left.\begin{array}{rccccc} & 1&3&5&7&9\\ & 7&3&9&5&1\\ \text{or} & 2&8&4&0&6\end{array}\right] \begin{array}{l}28_{12}\\68_{32}\end{array};\ \times \text{ by 3, add 4 or 9, and cast out the tens.}$$

$$\begin{array}{rrrrrr} & 1 & 3 & 5 & 7 & 9 \\ & 4 & 0 & 6 & 2 & 8 \\ \text{or} & 9 & 5 & 1 & 7 & 3 \end{array}\left]\begin{array}{l} 28_{62}; \\ 68_{82}, \end{array}\right. \times \text{ by 3, add 1 or 6, and cast out the tens.}$$

$$\begin{array}{rrrrrr} & 2 & 4 & 6 & 8 & 0 \\ & 0 & 6 & 2 & 8 & 4 \\ \text{or} & 5 & 1 & 7 & 3 & 9 \end{array}\left] 48_{72}; \right. \times \text{ by 3, add 4 or 9, and cast out the tens.}$$

Instead of adding a number its complement (10 — N) may be subtracted.

Instead of the odd numbers 1, 3, 5, 7, 9, put O, and instead of the even numbers 2, 4, 6, 8, 0 put E. We have then for cubes ending in 5 and their roots the following facts

$$\begin{array}{cc} 1 & 6 \\ E & O \end{array}\left]\begin{array}{l} 25_{05}, \\ 25_{45}; \\ 25_{85} \end{array}\right. \quad \begin{array}{cc} 1 & 6 \\ O & E \end{array}\left]\begin{array}{l} 25_{25}, \\ 25_{65}; \end{array}\right. \quad \begin{array}{cc} 3 & 8 \\ E & O \end{array}\left]\begin{array}{l} 75_{15}, \\ 75_{55}; \\ 75_{95} \end{array}\right. \quad \begin{array}{cc} 3 & 8 \\ O & E \end{array}\left]\begin{array}{l} 75_{35}, \\ 75_{75}. \end{array}\right.$$

It would be trespassing too much upon the limited space of the ANALYST to mention the beautiful harmonies displayed in these facts. I leave them for the reader to observe, hoping that some one may have the time and ingenuity to connect them in a mnemonic formula.

By means of the series preceding 32_{18} I detected an error in the table of cubes in Hutton's Mathematical Dictionary, Vol. 2. He gives 10,360,282 as the cube of 218 instead of 10,360,232.

NOTE ON THE SOUND HEARD IN CONNECTION WITH CONCHS AND OTHER SHELLS, BY GARVIN SHAW, Kemble, Ont., Canada.—When a conch, or any other shell of a similar construction is applied to the ear, a sound is generally heard resembling that of the waves at the sea shore. This phenomenon is produced by the successive reverberations of external sounds against the interior side of the conch. The external sound enters the conch, and is reflected from side to side until it has reached the extrmity of the winding cavity, and the noise which is heard is merely the continued echo.

That this explanation is true is manifest from the circumstance that the intensity of the sound in the conch varies with that of the external sound. If there be no noise whatever connected with the surrounding objects, there will be none in the conch, but if the external noise be great, the internal will also be considerable; nor is this effect connected solely with conchs or other shells of similar construction. If any hollow vessel be applied to the ear while there is an external sound, a continued noise will appear to issue from the vessel. The explanation therefore is true, and that it is also sufficient to explain the phenomenon is obvious.